Bayesian Blind Deconvolution with General Sparse Image Priors Supplementary Technical Note

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In the following, the equation and table numbers are denoted with preceding "A-". The ones without the preceding "A-" refer to the main paper.

Concave Conjugate Formulation

Given the function $\rho(\sqrt{x_\gamma(i)})$ which is concave and increasing on $(0,\infty)$, we have the concave conjugate pair

$$\rho\left(x_\gamma(i)\right) = \inf_{\xi_\gamma(i)>0} \frac{1}{2}\,\xi_\gamma(i)\,x_\gamma^2(i) \,-\, \rho^*\left(\frac{\xi_\gamma(i)}{2}\right) \tag{A-1}$$

$$\rho^*\left(\frac{\xi_\gamma(i)}{2}\right) = \inf_{x_\gamma(i)} \frac{1}{2}\,\xi_\gamma(i)\,x_\gamma^2(i) - \rho\left(x_\gamma(i)\right)\,. \tag{A-2}$$

Taking the derivative of the right hand side of (A-1) w.r.t. $\xi_\gamma(i)$ and setting it equal to zero, we obtain

$$x_\gamma^2(i) = \rho^{*'}\left(\frac{\xi_\gamma(i)}{2}\right)\,. \tag{A-3}$$

Next, taking the derivative of the right hand side of (A-2) w.r.t. $x_\gamma(i)$ and setting it equal to zero, we have

$$\frac{\rho'\left(x_\gamma(i)\right)}{x_\gamma(i)} = \xi_\gamma(i)\,. \tag{A-4}$$

The equalities in (A-3) and (A-4) specify the optimal value of $\xi_\gamma(i)$ for a specific value of $x_\gamma(i)$.

This representation is used in the paper in order to derive the cost-function for VB. Since the distribution $p(\xi_\gamma)$ is degenerate, using (A-3) in (A-4), the VB estimate of $\mathrm{E}\left[\xi_\gamma(i)\right] = \frac{\rho'(\nu_\gamma(i))}{\nu_\gamma(i)}$ in (23) can be written as the solution of

$$\mathrm{E}\left[\xi_\gamma(i)\right] = \arg\min_{\xi_\gamma(i)} \frac{1}{2}\,\xi_\gamma(i)\,\nu_\gamma^2(i) \,-\, \rho^*\left(\frac{\xi_\gamma(i)}{2}\right) \tag{A-5}$$

which is used in deriving the cost-function for the VB approach (26) in Section 3.2 of the main paper.